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Gravitational Waves in String theory in Anti-de Sitter Background

Alok Kumar*

Institute of Physics, Bhubaneswar 751 005, INDIA

ABSTRACT

Inspired by the studies of gravitational waves in anti-de Sitter universe, in general relativity, in this paper we investigate the possibility of similar solutions in IIB string theory on $AdS_3 \times S^3 \times R^4$. We give a general form for such solutions in this background and present several explicit examples, by directly solving the field equations, as well as the ones obtained by taking a scaling limit on $D1 - D5$ brane systems in a pp-wave background. The form of the metric in our solutions corresponds to a gravitational wave in AdS_3. We show the supersymmetric nature of these solutions and discuss the possibility of their generalizations to other anti-de Sitter backgrounds, including the ones in four dimensions.

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*kumar@iopb.res.in

Gravitational waves in general relativity, also known as pp-waves, have been a major research area for long time[1, 2]. They provide a geometrical framework to understand the gravitational radiation, in addition to having possible astrophysical relevance. However, space-times generally considered in these studies are asymptotically flat, the primary motivation being the analysis of radiation from finite sources. On the other hand, exact solutions representing gravitational wave in non-asymptotically flat backgrounds have also been a well-studied subject[3, 4, 5]. Such 'cosmological' gravitational wave solutions are expected to provide a useful model for the propagation of primordial gravitational waves.

Gravitational wave solutions in string theory have also been a subject of great interest from various points of view. They provide exact solutions of string equations of motion[6, 7] to all orders in inverse string tension, where the classical backgrounds do not receive worldsheet corrections. More recently, an exactly solvable string construction has been presented in a special class of such ten dimensional string backgrounds[8, 9], known as Hpp-waves, in the presence of constant flux for the five-form Ramon-Ramond (RR) field strength. Such configurations also appear in a 'Penrose' limit of $AdS^5 \times S^5$ string background[10] and have been argued to be 'dual' to four dimensional gauge theories, with $N = 4$ supersymmetry, in a sector with large R-charges[11, 12]. As a confirmation of the conjecture, exact anomalous dimensions of gauge theory operators and various other predictions of string theory, have been verified using gauge theory techniques as well. This correspondence between string and gauge theories, also known as BMN duality, led to a surge of activity in the studies of plane-wave solutions in last few years. Our particular interest in this paper will be in the classical D-brane solutions in plane-wave backgrounds[13, 14]. Plane-wave solutions considered in string theory, including those for BMN duality, however, have an interpretation of a wave structure in a flat Minkowski background space-time.

In this paper, we give, both the general structure, as well as, explicit solutions for gravitational waves in an anti-de Sitter background in string theory. More precisely, we analyze ten-dimensional supergravity equations, following from IIB string theory, in an $AdS_3 \times S^3 \times R^4$ background, with a standard R-R 3-form flux. For a suitable ansatz, we find out the conditions under which one has a gravitational wave solution in this background. Our result implies that, gravitational wave solutions can be generalized to string theory, by turning on suitable components of $NS - NS$ 3-form fluxes, in addition to the components of the metric, representing gravitational wave in AdS_3. Such components of metric and anti-symmetric tensor fields also appear in flat space pp-wave solutions. We then find out explicit examples of the gravitational waves in string theory, by first directly solving the conditions derived from the field equations in string theory. This solution has a wave structure only in the metric (since NS-NS three-form turns out to be absent in this case) and the role of the additional field, namely R-R three-form is to compensate for the $AdS_3 \times S^3$ curvature components.

We also obtain gravitational wave solutions by applying a scaling limit on the $D1 - D5$

brane solution in a pp-wave background[14]. The limiting solution satisfies the conditions for the existence of the gravitational waves in string theory on $AdS_3 \times S^3 \times R^4$. The supersymmetric nature of this example is already guaranteed by the fact that the $D1 - D5$ branes in the pp-wave background preserves a certain amount of supersymmetry. We, however, explicitly solve the Killing spinor equations and show that the solutions presented in this paper are supersymmetric. Several other gravitational wave solutions are also presented, by using S and T-duality transformations of string theory. We also discuss the possibility of the generalization of the results to four dimensional AdS space, by a suitable embedding in M-theory.

We now begin by writing down the ansatz for the gravitational wave solution in ten dimensional type IIB string theory on $AdS_3 \times S^3 \times R^4$. First, to write down the metric we use a structure which is similar to the one in the case of pure gravity theory in four dimensions[4], known as Siklos space-times:

$$ds^2 = q\left\{\frac{du^2}{u^2} + \frac{1}{u^2}(2dx^+dx^- + H(u,x^+)dx^{+2}) + d\Omega_3^2\right\} + \sum_{i=1}^{4} dx^{i2}, \tag{1}$$

with 'Poincare' coordinates x^+, x^- and u parameterizing AdS_3, when $H = 0$. The function H represents the gravitational wave profile. In our case, we assume the dependence of H only on u, in addition to x^+. This implies that gravitational wave polarization lies in the AdS_3 part only. Many interesting examples in both flat (such as Hpp-wave) as well as anti-de Sitter spaces (as in the case of certain Siklos type solutions in four dimensional gravity[4]) correspond to the case when H is independent of x^+. One of our example below, following from $D1-D5$ branes in a pp-wave background, also has a similar property. For the time being, however, we keep H more general, dependent both on u and x^+. In equation (1), coordinates x^i $(i = 1,..,4)$ represent R^4 coordinates and the metric on S^3 is parameterized as:

$$d\Omega_3^2 = d\theta^2 + cos^2\theta d\phi^2 + sin^2\theta d\psi^2. \tag{2}$$

In the following, indices a, b $(a = 1, 2, 3)$ etc. will also be used for denoting the coordinates on S^3. Full ten dimensional coordinates are denoted by Greek indices μ, ν etc.. The R^4 part of the metric does not play any crucial role in our discussion, however they are needed to saturate the critical dimension of the IIB string theory. Finally, the constant parameter q in equation (1) gives the size or radius of curvature for AdS_3 and S^3 spaces.

One also recalls that in string theory, the $AdS_3 \times S^3$ solutions are accompanied by appropriate NS-NS or R-R 3-form field strengths. To start with, we take these as the later (R-R) possibility. The R-R 3-form field strengths, in the coordinates that we are working, have the following form:

$$F^{(3)} = \pm\frac{2q}{u^3}dx^+ \wedge dx^- \wedge du \pm 2q \;\; sin\theta \;\; cos\theta \;\; d\theta \wedge d\phi \wedge d\psi \; . \tag{3}$$

We notice the presence of $\pm$ signs in front of both the terms in the above equation. They correspond to the fact that such a solution may originate from either $D1$ $(D5)$or $\bar{D}1$ $(\bar{D}5)$

branes. Equations of motion[15] are satisfied by all the possibilities, however, supersymmetry (in our notations below) implies a relative sign between these terms.

As is known, for $H = 0$, the metric and 3-form field strengths given in equations (1) and (3) already satisfy the supergravity field equations following from the type IIB string theory. For $H(u, x^+) \neq 0$ one needs to turn on other fields as well. The situation is in fact similar to that in flat space, where for example, the associated field strengths for the 3-form NS-NS fields are generally non-vanishing in directions transverse to the light-cone. In the case of flat space, a suitable choice[7] makes use of the field configurations, so that dilaton (ϕ) and NS-NS 3-form field strength $H^{(3)}$ take a form: $\phi \equiv \phi(x^+)$, $H^{(3)}_{+IJ} = A_{IJ}(x^+)$, with I, J running over the transverse directions. With certain constraint, these fields, together with the flat space pp-wave metric, satisfy the field equations.

Guided by the general form of the NS-NS 3-form field strengths in the previous paragraph, for flat space pp-wave, we now propose the following form for them in the $AdS_3 \times S^3 \times R^4$ background:

$$H^{(3)} = dx^+ \wedge \left((\frac{q^2 A_a(x^a, x^+)}{u^3}) du \wedge dx^a + (\frac{q^2 B_{ab}(x^a, x^+)}{u^2}) dx^a \wedge dx^b \right). \tag{4}$$

The form of $H^{(3)}$ in our case corresponds to the situation when (as in flat space) non-zero components have been turned on for $H_{+\mu\nu}$'s along all the directions of $AdS_3 \times S^3$ that are transverse to the light-cone. However, now there is a non-trivial dependence of the components of $H_{+\mu\nu}$ on the transverse coordinates as well. In equation (4), we have already identified the dependence on one of the coordinates transverse to the light-cone, namely u. The dependence of A_a's and B_{ab}'s on S^3 coordinates will be determined later on from the field equations. The powers of u in the coefficients of A_a and B_{ab} in equation (4) are also dictated by the field equations which will be discussed below.

For the gravitational wave solution that we are discussing, the metric in equation (1), R-R 3-form field strengths in equation (3) and NS-NS 3-form field strengths in equation (4) are the only non-vanishing fields. All other fields of IIB theory, including the dilaton are taken to be zero. Further restriction on the dilaton field in the present case, unlike the one in flat space as mentioned above, comes from a dilaton dependent factor in front of the contribution of the R-R fields in the gravitational field equation of the IIB theory.

The non-zero components of the Ricci tensor ($R_{\mu\nu}$) along directions $x^\pm, u$, for our choice of the metric in equation (1), are:

$$R_{++} = -\frac{H_{,uu}}{2} + \frac{1}{2}\frac{H_{,u}}{u} - \frac{2H}{u^2}, \quad R_{+-} = -\frac{2}{u^2}, \quad R_{uu} = -\frac{2}{u^2} \,. \tag{5}$$

Other non-zero components of the Ricci tensor are along S^3 directions:

$$R_{\theta\theta} = 2, \quad R_{\phi\phi} = 2cos^2\theta, \quad R_{\psi\psi} = 2sin^2\theta. \tag{6}$$

All the type IIB field equations are satisfied for our general ansatz in equations (1), (3) and (4), provided a set of conditions are satisfied by functions $H(u, x^+)$, A_a's and B_{ab}'s. The condition following from the field equation of the metric component g_{++} is:

$$-\frac{H_{,uu}}{2} + \frac{1}{2}\frac{H_{,u}}{u} = \frac{q^2}{u^4}\left(\frac{1}{2}A_a A^a + \frac{1}{4}B_{ab}B^{ab}\right), \tag{7}$$

where the raising and lowering of the indices in A_a and B_{ab} has been done with respect to the metric on S^3 †. The similarity of the above condition (7) can also be seen with anti-de Sitter gravitational waves in four dimensional gravity[4], by setting A_a's and B_{ab}'s to zero.

The field equations for 3-form fields also imply the following conditions on quantities A_a's and B_{ab}'s:

$$\nabla^a A_a = 0, \quad A_a + \frac{1}{2}\nabla^b B_{ab} = 0, \tag{9}$$

with covariant derivatives being defined by the metric on S^3 given in equation (2). Finally, the Bianchi identity of H gives a restriction:

$$\partial_a A_b - \partial_b A_a = -2B_{ab}. \tag{10}$$

We have therefore presented the general structure of gravitational waves in type IIB string theory in $AdS_3 \times S^3 \times R^4$ background. They are characterized by functions H, A_a and B_{ab} which satisfy conditions (7), (9) and (10). We now give explicit solutions for these conditions for two different choices of H, A_a and B_{ab}'s.

First, a solution can be obtained by considering $A_a = B_{ab} = 0$, so that NS-NS three-form field $H^{(3)}$ is trivial. Then one only has a non-trivial R-R three-form flux given by equation (3), in addition to the metric (1). Moreover, the R-R three-form flux is identical to the one in the case of $AdS_3 \times S^3$ background. This choice of A_a and B_{ab} already satisfies equations (9) and (10). Equation (7) in this case implies a general wave profile given by

$$H(u, x^+) = f(x^+)u^2 + g(x^+), \tag{11}$$

with f and g being functions of x^+ only. Later on we will also discuss the supersymmetry property of this solution.

We now write down another gravitational wave solution by taking a scaling limit on a known $D1-D5$ brane in a pp-wave background[14]. In this case, the gravitational wave profile turns out to be independent of x^+ (similar to the case of Hpp-waves appearing in the case of 'BMN' duality), although further generalization of this solution to include x^+ dependence is possible and will also be discussed below.

†Analogous condition for the pp-wave solution in flat space, mentioned earlier, has a form[7]:

$$\partial^I \partial_I H(x^+, x^I) + A_{IJ}A^{IJ} + 4\partial^2_{x^+}\phi(x^+) = 0 \tag{8}$$

In fact, it can be verified that the choice of A_a components:

$$A_\theta = 0, \;\; A_\phi = 2\mu cos^2\theta, \;\; A_\psi = 2\mu sin^2\theta, \tag{12}$$

and nonzero B_{ab} components:

$$B_{\theta\phi} = \mu sin 2\theta, \;\; B_{\theta\psi} = -\mu sin 2\theta, \tag{13}$$

satisfy equations (9) and (10). The parameter μ in the above equation characterizes the gravitational wave. When $\mu = 0$, one reduces to $AdS_3 \times S^3$. Finally, equation (7) is also satisfied for:

$$H = -\frac{\mu^2 q^2}{u^2}. \tag{14}$$

Equations (12), (13) and (14), together with the metric and 3-form fields in equations (1), (3) and (4), give a gravitational wave solution in type IIB string theory on $AdS_3 \times S^3 \times R^4$. We now show the connection of this solution to the $D1 - D5$ branes in a pp-wave background in type IIB string theory.

The $D1 - D5$ brane solution, reducing to the above gravitational wave on $AdS_3 \times S^3 \times R^4$ in a scaling limit, is given as [14]:

$$\begin{aligned}
ds^2 &= (f_1 f_5)^{-\frac{1}{2}} (2dx^+ dx^- - \mu^2 \sum_{i=1}^{4} x_i^2 (dx^+)^2) + \Big(\frac{f_1}{f_5}\Big)^{\frac{1}{2}} \sum_{a=5}^{8} (dx^a)^2 \\
&+ (f_1 f_5)^{\frac{1}{2}} \sum_{i=1}^{4} (dx_i)^2, \\
e^{2\phi} &= \frac{f_1}{f_5,} \\
H^{(3)}_{+12} &= H^{(3)}_{+34} = 2\mu, \\
F^{(3)}_{+-i} &= \partial_i {f_1}^{-1}, \qquad F^{(3)}_{mnp} = \epsilon_{mnpl} \partial_l f_5,
\end{aligned} \tag{15}$$

where f_1 and f_5 are the Green functions, in common transverse directions $x^1, .., x^4$, representing $D1$ and $D5$ branes respectively: $f_1 = 1 + \frac{q_1}{r^2}$, $f_5 = 1 + \frac{q_5}{r^2}$ and r is the radial coordinate in the transverse direction. In the scaling limit one takes (see for example[16]): $r \longrightarrow 0$, in addition to setting $q_1 = q_5 = q$. It can now be verified that such a scaling limit, together with a coordinate change $r = \frac{q}{u}$, gives the gravitational wave solution of equations (1), (3) and (4) when the wave profile is chosen as in equations (12), (13) and (14). We have therefore shown that $D1 - D5$ brane solution in pp-wave background gives, in a scaling limit, a gravitational wave in string theory in $AdS_3 \times S^3 \times R^4$ background.

We now discuss the supersymmetry properties of our solutions, first for the specific wave profile in equations (12), (13) and (14). Since this specific solution is obtained, in a scaling

limit, from a known BPS solution which already preserves a fraction of supersymmetry, we expect it to be supersymmetric as well. Nevertheless, an explicit analysis is carried out below to find out the Killing spinors.

Killing spinor equations follow from the dilatino and gravitino variations of the ten dimensional type IIB supergravity[17]. To write these equations, we make use of the form of the S^3 metric given in equation (2). Then, in order to satisfy the dilatino variation conditions, $\delta\lambda_\pm = 0$, one needs to impose the following conditions on the ten-dimensional supersymmetry parameters, given by Majorana-Weyl spinors $\epsilon_\pm$ with positive chirality, and $\pm$ denote the left and the right moving sectors of the IIB string theory. The conditions are:

$$\Gamma^{\hat{+}}\epsilon_\pm = 0, \tag{16}$$

$$(1 \pm \Gamma^{\hat{+}\hat{-}\hat{u}\hat{\theta}\hat{\phi}\hat{\psi}})\epsilon_\pm = 0, \tag{17}$$

where $\Gamma^{\hat{+}}$, $\Gamma^{\hat{-}}$ etc. are the Dirac gamma matrices, with superscripts denoting the tangent space indices. The $\pm$ signs in equation (17) correspond to the choice of the relative signature in equation (3).

Variations of gravitino imply several conditions depending on the components $\delta\psi_\mu$. In order to satisfy them, and solve these equations, one needs to impose:

$$\Gamma^{\hat{u}\hat{\theta}\hat{\phi}\hat{\psi}}\epsilon_\pm = \epsilon_\pm, \tag{18}$$

in addition to equations (16) and (17). Now, the consistency between equations (16), (17) and (18) [‡] implies that one needs to pick up the lower sign in equation (17), in order to preserve supersymmetry.

To clarify further, equation (17) is the 1/2 supersymmetry condition for the $AdS_3 \times S^3 \times R^4$ background in the absence of a gravitational wave. The other two conditions, namely (16) and (18) arise when gravitational wave, with the particular profile mentioned above in equations (12) - (14), is present. By using these two conditions, all the Killing spinor equations, appearing as first order differential equations, reduce precisely to the case of pure $AdS_3 \times S^3 \times R^4$ background. Since these equations are known to admit the maximal number of Killing spinors, residual supersymmetry is obtained by analyzing the projection conditions: (16), (17) and (18) on their solutions.

Explicitly, the Killing spinor equations reduce to the following simple forms by imposing the supersymmetry conditions coming from the presence of the gravitational waves, namely equations (16) and (18). By defining $\epsilon = \epsilon_+ + \epsilon_-$, $\tilde{\epsilon} = \epsilon_+ - \epsilon_-$ we get:

$$\partial_u\epsilon + \frac{1}{2u}\Gamma^{\hat{+}\hat{-}}\epsilon = 0, \;\; \partial_-\epsilon - \frac{1}{u}\Gamma^{\hat{+}\hat{u}}\epsilon = 0, \;\; \partial_+\epsilon = 0, \tag{19}$$

[‡] where we have used the fact that, by using a choice of the metric: $\{\Gamma^{\hat{+}}, \Gamma^{\hat{-}}\} = 2$, condition $\Gamma^{\hat{+}}\epsilon_\pm = 0$ is equivalent to $\Gamma^{\hat{+}\hat{-}}\epsilon_\pm = \epsilon_\pm$.

$$\partial_u \tilde{\epsilon} - \frac{1}{2u}\Gamma^{\hat{+}\hat{-}}\tilde{\epsilon} = 0, \;\; \partial_- \tilde{\epsilon} = 0, \;\; \partial_+ \tilde{\epsilon} - \frac{1}{u}\Gamma^{\hat{-}\hat{u}}\tilde{\epsilon} = 0, \tag{20}$$

and

$$\partial_\theta \epsilon - \frac{1}{2}\Gamma^{\hat{\phi}\hat{\psi}}\epsilon = 0, \;\; \partial_\phi \epsilon + \frac{sin\theta}{2}\Gamma^{\hat{\theta}\hat{\phi}}\epsilon + \frac{cos\theta}{2}\Gamma^{\hat{\theta}\hat{\psi}}\epsilon = 0, \;\; \partial_\psi \epsilon - \frac{cos\theta}{2}\Gamma^{\hat{\theta}\hat{\psi}}\epsilon - \frac{sin\theta}{2}\Gamma^{\hat{\theta}\hat{\phi}}\epsilon = 0, \tag{21}$$

$$\partial_\theta \tilde{\epsilon} + \frac{1}{2}\Gamma^{\hat{\phi}\hat{\psi}}\epsilon = 0, \;\; \partial_\phi \tilde{\epsilon} + \frac{sin\theta}{2}\Gamma^{\hat{\theta}\hat{\phi}}\tilde{\epsilon} - \frac{cos\theta}{2}\Gamma^{\hat{\theta}\hat{\psi}}\tilde{\epsilon} = 0, \;\; \partial_\psi \tilde{\epsilon} - \frac{cos\theta}{2}\Gamma^{\hat{\theta}\hat{\psi}}\tilde{\epsilon} + \frac{sin\theta}{2}\Gamma^{\hat{\theta}\hat{\phi}}\tilde{\epsilon} = 0. \tag{22}$$

The set of above equations have simple solutions for both ϵ and $\tilde{\epsilon}$. They are:

$$\epsilon = (e^{\frac{1}{2}\Gamma^{\hat{\phi}\hat{\psi}}\theta}e^{-\Gamma^{\hat{\theta}\hat{\psi}}(\phi-\psi)})(e^{-\frac{1}{2}\Gamma^{\hat{+}\hat{-}}lnu}e^{\Gamma^{\hat{+}\hat{u}}x^-})\epsilon_0, \tag{23}$$

and

$$\tilde{\epsilon} = (e^{-\frac{1}{2}\Gamma^{\hat{\phi}\hat{\psi}}\theta}e^{\Gamma^{\hat{\theta}\hat{\psi}}(\phi+\psi)})(e^{\frac{1}{2}\Gamma^{\hat{+}\hat{-}}lnu}e^{\Gamma^{\hat{-}\hat{u}}x^+})\tilde{\epsilon}_0, \tag{24}$$

where ϵ_0 and $\tilde{\epsilon}_0$ are constant spinors.

Now, to count the amount of supersymmetry that is preserved, we notice that the supersymmetry projection (17) commutes with all the operators appearing in the exponents in expressions (23) and (24). As a result, this projection gives a condition identical to (17), when acting on ϵ_0 and $\tilde{\epsilon}_0$. This is the 1/2 supersymmetry in the absence of the gravitational wave.

The remaining projections, (16) and (18) on spinor ϵ also lead to identical conditions on ϵ_0. This, however, does not hold when considering the spinor $\tilde{\epsilon}$ in equation (24) due to the presence of $\Gamma^{\hat{-}}$. One is therefore left with only a Killing spinor ϵ in the presence of the gravitational wave. This spinor is also further restricted by two independent projections (16) and (17). Thus we finally have 1/8 supersymmetry for our solution.

Several other gravitational wave solutions in $AdS_3 \times S^3 \times R^4$ background can be constructed using 'duality' symmetries of string theory. The simplest ones of these comes from a direct application of S-duality on our general solution in equations (1), (3) and (4). This symmetry leaves the metric unchanged, however, NS-NS and R-R 3-form fields are interchanged among themselves under the symmetry transformation. The specific wave profiles of both the solutions mentioned in the paper, namely the ones in (11) and (12)-(14), remain unchanged under the symmetry transformation. We do not write these solutions explicitly.

A more non-trivial example involves several applications of S and T-dualities, amounting to taking a scaling limit on another $D1 - D5$ brane solution[14], involving now self-dual R-R 5-form field strengths ($F^{(5)}$) of the IIB string theory, rather then NS-NS 3-forms $H^{(3)}$ as in equation (15). The gravitational wave solution obtained by taking a scaling limit has the identical metric and 3-form flux as in equations (1) and (3). However, one now has a (self-dual) R-R 5-form flux (in place of NS-NS 3-form of equation (4)):

$$F^{(5)} = dx^+ \wedge \left((\frac{q^2 A_a(x^a, x^+)}{u^3})du \wedge dx^a + (\frac{q^2 B_{ab}(x^a, x^+)}{u^2})dx^a \wedge dx^b \right) \wedge (dx^1 \wedge dx^2 + dx^3 \wedge dx^4), \tag{25}$$

and, as already mentioned earlier, $x^1, .., x^4$ are the coordinates of R^4. The wave profile for this gravitational wave solution is identical to the one in (12), (13) and (14).

Further generalization of the gravitational wave solution, as obtained above from the $D1-D5$ branes in equations (12) - (14), can be incorporated in string theory. It is evident from equations (7), (9) and (10) that one can obtain a general class of explicit solutions by multiplying H in equation (14) by any function $F(x^+)$, provided A_a and B_{ab} in equations (12) and (13) are multiplied by another function $G(x^+)$, satisfying $F(x^+) = G(x^+)^2$. Supersymmetry analysis for these x^+ dependent solutions proceeds in a similar fashion as presented above. The amount of supersymmetry also turns out to be identical. The primary reason for the similarity in the supersymmetry analysis is the fact that the supersymmetry projections coming from the gravitational waves, namely equations (16) and (18), still remain valid and reduce the Killing spinor equations once again to the same ones as in equations (19) - (22). One also has an identical amount of supersymmetry for another x^+ dependent solution presented in equation (11). The difference in the supersymmetry analysis, in this case, is the absence of the projection (18). However, since this condition is not an independent one, as discussed earlier, the amount of supersymmetry remains same.

We have therefore presented explicit solutions in string theory representing gravitational waves in AdS_3. It is possible to generalize our results in various ways. First, it may be possible to use other branes in pp-wave backgrounds, in order to obtain gravitational waves in various other anti-de Sitter spaces in string theory. In particular, in our view, an interesting possibility is to look for gravitational waves in AdS_4, through a suitable solution in $AdS_4 \times S^7$ background of M-theory. This can possibly be done by using an $M2$ brane solution in a pp-wave background, in the same way as above. Alternatively, one may be able to solve the M-theory equations of motion directly, with a suitable ansatz, like the one given in equations (1), (3) and (4) for AdS_3. Such solutions will provide the generalization of 'Siklos space-times', discussed in pure gravity, to M-theory. Another interesting aspect will be to examine if any of the gravitational wave solutions presented here provide exact string solutions, like the pp-waves or some $AdS_3 \times S^3$ backgrounds. It will also be useful to find out the connection of our results to certain shock wave solutions found in [18].

Note Added: After the submission of this paper to the archive, I have also come to know of some other papers[19, 20] where gravitational waves in anti-de Sitter spaces have been discussed. However, these gravitational waves correspond, in our language, to the situation when $A_a = B_{ab} = 0$ in our equations (4) and (25). In particular, our $A_a = B_{ab} = 0$ solution (11) has been given earlier in [19] and shown to be equivalent to the pure AdS_3 without a gravitational wave. However, it is emphasized that the general situation above represents new gravitational wave structure in anti-de Sitter backgrounds with a non-trivial NS-NS 3-form (or R-R 5-form) flux mixing AdS_3 and S^3 spaces.

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